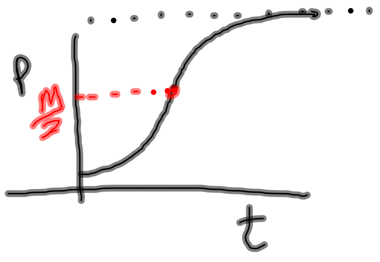
6.5 logistic growth



M = carrying capacity

$$\frac{dP}{dt} = kP(M-P)$$

de for logistic growth

1. for what value of P is the population growing fastest?

$P = \frac{M}{2}$

2. For what values of P is the rate of change of P about zero?

$P = M$, $t \to \infty$; $P = \cancel{0}$ P_0, $t = 0$

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How to solve $\frac{dP}{dt} = kP(M-P)$

find $P(t)$

bears $P_0 = 10$ $\frac{dP}{dt} = .008\,P(100-P)$

sep. var. $\frac{dP}{P(100-P)} = .008\,dt$

integrate $\int \frac{1}{P(100-P)}\,dP = \int .008\,dt$

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Partial fractions

$$\frac{1}{P(100-P)} = \frac{(100-P)}{(100-P)}\frac{A}{P} + \frac{B}{(100-P)}\frac{P}{P}$$

A, B are constants

$$\frac{1}{P(100-P)} = \frac{100A - AP + BP}{P(100-P)}$$

$$1 + 0P = 100A + (-A+B)P$$

$$-A+B=0 \qquad A = \frac{1}{100}$$

$$100A = 1 \qquad B = \frac{1}{100}$$

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$$\int \frac{1}{P(100-P)}\,dP = \int .008\,dt$$

$$\int \frac{1/100}{P} + \frac{1/100}{100-P}\,dP = \int .008\,dt$$

$$\frac{1}{100}\int \frac{1}{P} + \frac{1}{100-P}\,dP = \int .008\,dt$$

$$\frac{1}{100}\left(\ln(P) + -\ln(100-P)\right) = .008t + C$$

solve for P

$$e^{\ln\left(\frac{P}{100-P}\right)} = e^{(.8t + C)}$$

flip both sides

$$\frac{P}{100-P} = e^{.8t}\cdot e^{C} = Ce^{.8t}$$

$$\frac{100-P}{\cancel{P}} = \frac{1}{Ce^{.8t}}\cdot P$$

factor P: $100 = \frac{1}{Ce^{.8t}}\cdot P + P$

$$100 = P\left(\frac{1}{Ce^{.8t}} + 1\right)$$

$$\frac{100}{1+\frac{1}{C}e^{-.8t}} = \frac{100}{\frac{1}{Ce^{.8t}}+1} = P$$

$A = \frac{1}{C}$ $\qquad P = \frac{100}{1+Ae^{-.8t}}$

$$A = \frac{100-P_0}{P_0}$$

if $P_0 = 10$ $\qquad A = 9$

$$P = \frac{\overbrace{100}^{M}}{1+\underbrace{9}_{A}e^{\overbrace{-.8t}^{k\cdot M}}}$$

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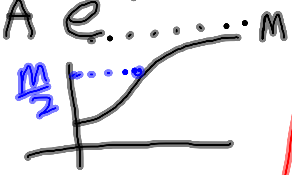
general logistic de with solution

de: $\frac{dP}{dt} = k \cdot P(M - P)$

solution: $P = \frac{M}{1 + Ae^{-kMt}}$

$A = \frac{M - P_0}{P_0}$



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Ex 5 $P_0 = 61$ $\frac{dP}{dt} = .0003\,P(1000 - P)$

Soln: $P = \frac{1000}{1 + 15.3934\,e^{-.3t}}$

$A = \frac{1000 - 61}{61}$

$A = 15.3934$

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